

Short-Term Variations in the Equatorial Rotation Rate of Sunspot Groups

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Abstract We have detected several periodicities in the solar equatorial rotation rate of sunspot groups in the Greenwich Photoheliographic Results (GPR) during the period 1931–1976, the *Solar Optical Observing Network* (SOON) during the period 1977–2014, and the Debrecen Photoheliographic Data (DPD) during the period 1974–2014. We have compared the results from the fast Fourier transform (FFT), the maximum entropy method (MEM), and from Morlet wavelet power-spectra of the equatorial rotation rates determined from SOON and DPD sunspot-group data during the period 1986–2007 with those of the Mount Wilson Doppler-velocity data during the same period determined by Javaraiah *et al.* (2009, **257**, 61). We have also compared the power-spectra computed from the DPD and the combined GPR and SOON sunspot-group data during the period 1974–2014 to those from the GPR sunspot-group data during the period 1931–1973. Our results suggest a ~ 250 -day period in the equatorial rotation rate determined from both the Mt. Wilson Doppler-velocity data and the sunspot-group data during 1986–2007. However, a wavelet analysis reveals that this periodicity appears mostly around 1991 in the velocity data, while it is present in most of the solar cycles covered by the sunspot-group data, mainly near the minimum epochs of the solar cycles. We also found the signature of a period of ~ 1.4 years period in the velocity data during 1990–1995, and in the equatorial rotation rate of sunspot groups mostly around the year 1956. The equatorial rotation rate of sunspot groups reveals a strong ~ 1.6 -year periodicity around 1933 and 1955 and a weaker one around 1976, and a strong ~ 1.8 -year periodicity around 1943. Our analysis also suggests periodicities of ~ 5 years, ~ 7 years, and ~ 17 years as well as some other short-term periodicities. However, short-term periodicities are mostly present at the time of solar minima. Hence, short-term periodicities cannot be confirmed because of the larger uncertainty in the data.

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1. Introduction

In addition to the well-known 11-year solar cycle, solar activity varies on many shorter and longer timescales. For example, periodicities shorter than a year and of about two years are found in many solar activity indices (Rieger *et al.*, 1984; Lean and Brueckner, 1989; Pap, Bouwer, Tobiska, 1990; Bai and Sturrock, 1991; Bouwer, 1992; Richardson *et al.*, 1994; Krivova and Solanki, 2002; Özgüç, Atac, and Rybák, 2003; Kane, 2003; Bai, 2003; Obridko and Shelting, 2000; Obridko and Shelting, 2007; Chowdhury, Khan, and Ray, 2009; Scafetta and Willson, 2013; Kilcik *et al.*, 2014; Chowdhury *et al.*, 2016; and references therein). Studies of similar variations in the solar rotation data may help us to better understand the physical processes responsible for the solar variability. Recently, Javaraiah (2013) determined solar cycle variations in the mean equatorial rotation rate of the sunspot groups, with and without the data of abnormal angular motions of the sunspot groups. He found a large difference between the solar cycle variations in the yearly mean values of the equatorial rotation rates determined from the Mt. Wilson Doppler-velocity data and the sunspot-group data that did not include the abnormal motions. The patterns of the solar cycle variations of the equatorial rotation rate determined from the sunspot-group data that included the abnormal angular motions of the sunspot groups and the Mt. Wilson Doppler-velocity data closely resemble each other. Earlier, Javaraiah and Komm (1999) analyzed the Mt. Wilson Doppler-velocity data during 1986–1994 and found a ~ 1.2 -year and a few other short-term periodicities in the mean solar rotation rate. A study of the same data by Javaraiah (2011) found a few quasi-periodicities, ranging from a few days to a month, in the solar differential rotation rate. These short-term periodicities in the solar surface equatorial rotation rate were also found by Javaraiah *et al.* (2009), using the corrected Mt. Wilson Doppler-velocity measurements during the period 1986–2007. Here we investigate the possible short-term periodicities in the mean equatorial rotation rate of sunspot-group data and compare these periodicities to those found in the equatorial rotation rate determined from the Doppler-velocity data. Because of some inconsistencies in the rotational results obtained from the sunspot data measured at different observatories (Javaraiah, Bertello, and Ulrich, 2005, and references therein), here we use three sets of sunspot-group data. The Mt. Wilson Doppler-velocity data were acquired from December 1986 to March 2007, during Solar Cycles 22 and 23, while the sunspot data are available for a much longer period of time.

The data analysis is described in the next section, while in Section 3 we discuss the results from the spectral and wavelet analysis of the Mt. Wilson Doppler-velocity data and the three sets of sunspot-group data. A comparison of power spectra with those previously determined from the Mt. Wilson Doppler-velocity data by Javaraiah *et al.* (2009) is also shown. The summary and discussion of these results is given in Section 4.

2. Data Analysis

The solar differential rotation can be determined from the full-disk velocity data using the standard polynomial expansion

$$\omega(\phi) = A + B \sin^2 \phi + C \sin^4 \phi, \quad (1)$$

while for sunspot data, which are confined to only low and medium latitudes, it is sufficient to use the first two terms of the expansion, *i.e.*

$$\omega(\phi) = A + B \sin^2 \phi, \quad (2)$$

where $\omega(\phi)$ is the solar sidereal angular velocity at latitude ϕ , the coefficient A represents the equatorial rotation rate and B and C measure the latitudinal gradient in the rotation rate, B is associated mainly with low latitudes and C is associated largely with higher latitudes.

Here we use the Greenwich Photoheliographic Results, GPR (1931–1976), SOON (1977–2014), and Debrecen Photoheliographic Data, DPD (1974–2014), sunspot-group data. The GPR and SOON data were taken from the website <http://solarcience.msfc.nasa.gov/greenwich.shtml>, and the DPD data from <http://fenyi.solarobs.unideb.hu/pub/DPD/>. For each daily sunspot-group observation, the files include information about the time of the observation, the heliographic latitude (ϕ) and longitude (L), and the central meridian distance (CMD).

The GPR data have been compiled from the majority of the white-light photographs stored at the Royal Greenwich Observatory and at the Royal Observatory at the Cape of Good Hope. The gaps in these observations were filled with photographs from other observatories, such as Kodaikanal Observatory, India, the Hale Observatory, California, and the Heliophysical Observatory at Debrecen, Hungary. The Royal Greenwich Observatory terminated the publication of GPR at the end of 1976. Since 1977 the Debrecen Heliophysical Observatory took over this task. The SOON data included measurements made by the United States Air Force (USAF) from the sunspot drawings of a network of observatories that included telescopes in Boulder, Colorado, Hawaii, *etc.* David Hathaway scrutinized the GPR and SOON sunspot-group data and produced a reliable continuous data series from 1874 until today (Hathaway *et al.*, 2003; Hathaway and Choudhary, 2008; Hathaway, 2015). The DPD contain the positions and areas of sunspots, the total area and the mean positions of the sunspot groups, for each day compiled by using white-light full-disk observations taken at the Heliophysical Observatory, Debrecen, Hungary, and its Gyula Observing Station as well as at some other observatories. When no ground-based observation was found, space-borne quasi-continuum images obtained by the *Michelson Doppler Imager* (MDI) onboard the Solar and Heliospheric Observatory (SOHO) were used (for details see Györi *et al.*, 2010).

The determination of the sidereal rotation rates of the sunspot groups is briefly described in Javaraiah (2013). The ratio of the difference (ΔL) between the values of heliographic longitudes to the difference (Δt) between the times of the consecutive day observations of the sunspot groups is first computed.

Then, the value of the Carrington rigid-body rotation rate ($14.18^\circ \text{ day}^{-1}$) is added to this ratio. Javaraiah and co-workers have used this method in most of their earlier studies of solar rotation rate determined from sunspot-group data (Javaraiah, Bertello, and Ulrich, 2005; Javaraiah, 2013, and references therein). The data (the values of ω and mean ϕ of the consecutive days) are fitted to Equation (2) to obtain the values of A and B coefficients. In this article we mainly intend to compare the power spectra of the equatorial rotation rate $[A]$ derived from the sunspot-group data with the corresponding spectra derived from the Mt. Wilson Doppler-velocity data by Javaraiah *et al.* (2009). The daily values of A determined from the corrected (for scattered light, *etc.*, Ulrich, 2001) Mt. Wilson Doppler-velocity measurements cover the time interval from 1986 to 2007. In an earlier analysis (Javaraiah *et al.*, 2009) of this data set, large spikes, *i.e.* the values $> 2\sigma$ (where σ is the standard deviation), were removed from the daily data.

The Mt. Wilson Doppler-velocity daily data set contains several missing days, with gaps as long as 49 days. Javaraiah *et al.* (2009) binned these data in 61-day time intervals to produce a time series without gaps that is suitable for the required spectral analysis. Here we use a similar approach for the sunspot-group data by binning them into 61-day time intervals. First we analyzed the SOON and DPD sunspot-group data during 1986–2007, *i.e.*, for the same period for which the Mt. Wilson velocity-data are available. The coalignment in time of the sunspot-group data with the velocity data produced 126 61-day samples. Both the DPD sunspot-group data and the combined GPR and SOON (GPR-SOON) data are available for the period 1974–2014. We compared the power spectra of the A times series determined from these two data sets and also analyzed the GPR sunspot-group data from <http://fenyi.solarobs.unideb.hu/pub/DPD/> for the period 1931–1973. GPR sunspot-group data before 1931 were not used because of their poor quality during periods of solar cycle minima.

We did not use the sunspot-group data with $|CMD| > 75^\circ$ on any day of the sunspot-group life time. This reduces the foreshortening effect (if any). However, in this analysis the abnormal motions of the sunspot groups, *i.e.* the data corresponding to $\delta L > 3^\circ \text{ day}^{-1}$, were included. This increases the amount of data in a given 61-day interval, particularly during the solar cycle minima. In addition, these data were found to agree quite well with the Doppler-velocity data (Javaraiah, 2013). Equation (2) was used to derive the values of A from the sunspot-group data, and then the FFT, MEM, and Morlet wavelet power spectra of the A time series were computed. The MEM FORTRAN code was provided by A. V. Raveendran, which was also used in the earlier papers (Javaraiah and Gokhale, 1995; Javaraiah and Gokhale, 1997a; Javaraiah *et al.*, 2009). ■

Both FFT and MEM spectral methods are well-known techniques. In particular, MEM uses a parametric modeling approach to estimate the power spectrum of a time series. The method is data adaptive because it is based upon an autoregressive (AR) modeling process. An AR process is predictive; any point (after the first) is calculated by a linear combination of M (order of the process) previous values. An important step in this method is the optimal selection of the order M . If M is chosen too small, then the model smooths the data excessively and the resulting estimate of the power spectra is poorly resolved. If M is chosen

to be too large, then frequency shifting and spontaneous splitting of peaks can occur (see Ulrych and Bishop, 1975, and references therein).

Before the FFT is computed, the mean value is subtracted from the data, and the time series tapered by multiplying the first and the last 10% of the data by a half cosine bell (Brault and White, 1971). We have padded the time series with zeros so that the number of data points $[N]$ corresponds to an exact power of two (we have added two zeros to the data during 1986–2007). The significance levels of the peaks in the FFT spectra are computed by using both white-noise and red-noise models (see Torrence and Compo, 1998). The MEM code that we have used here takes the values for M in the range $[N/3, N/2]$ (Ulrych and Bishop, 1975) or $2N/\ln(2N)$ (Berryman, 1978). We adjusted the value of M until the FFT and MEM spectra showed a good agreement with each other. We found that $M = N/3$ is suitable for our analysis, producing spectra with peaks that are considerably sharp and well separated.

The wavelet transform is particularly effective for analyzing non-stationary signals. It can detect transient periodic signals and track their amplitude variation in time, as is typical of most solar phenomena. Here we applied the Morlet wavelet analysis (Torrence and Compo, 1998) to the A time series.

3. Results

The upper panel of Figure 1 shows the variations in A determined from the Mt. Wilson velocity-data during 1986–2007. The middle and lower panels show the corresponding variations determined from SOON and DPD sunspot-group data for the same period. Both median and standard deviation values were determined for each time series and were used to identify outliers. Values of A whose σ values exceed 2.6 times the corresponding median value were replaced with average of the corresponding values and their respective two neighbors. The solid curve represents the corrected data and the original data points are connected by the dotted curve. The three plots in Figure 1 show variations on different timescales, ranging from a few months to several years, in the A coefficient determined from the velocity and the sunspot-group data. However, the correlation between the velocity data and the sunspot-group data is only -6% to -8%, while the correlation between the variations in A determined from SOON and DPD sunspot-group data is much higher, around 33%. The large variation in the velocity data during 1990–1995 may be caused by several changes in the Mt. Wilson spectrometer that occurred during this period (see Javaraiah *et al.*, 2009). Figures 2, 3, and 4 show the corresponding FFT, MEM, and Morlet wavelet spectra of the data. In the wavelet spectra, the cross-hatched regions indicate the cone of influence where the edge effects become significant and the signal is statistically unreliable (Torrence and Compo, 1998). The dashed contours show the power above the 95% confidence level. The FFT and MEM spectra of the velocity data shown here are slightly different from those discussed in Javaraiah *et al.* (2009). For the reasons discussed in Section 1, the high-frequency peaks that were detected in Javaraiah *et al.* (2009) are much more reduced in the present spectra. However, the overall properties of the present

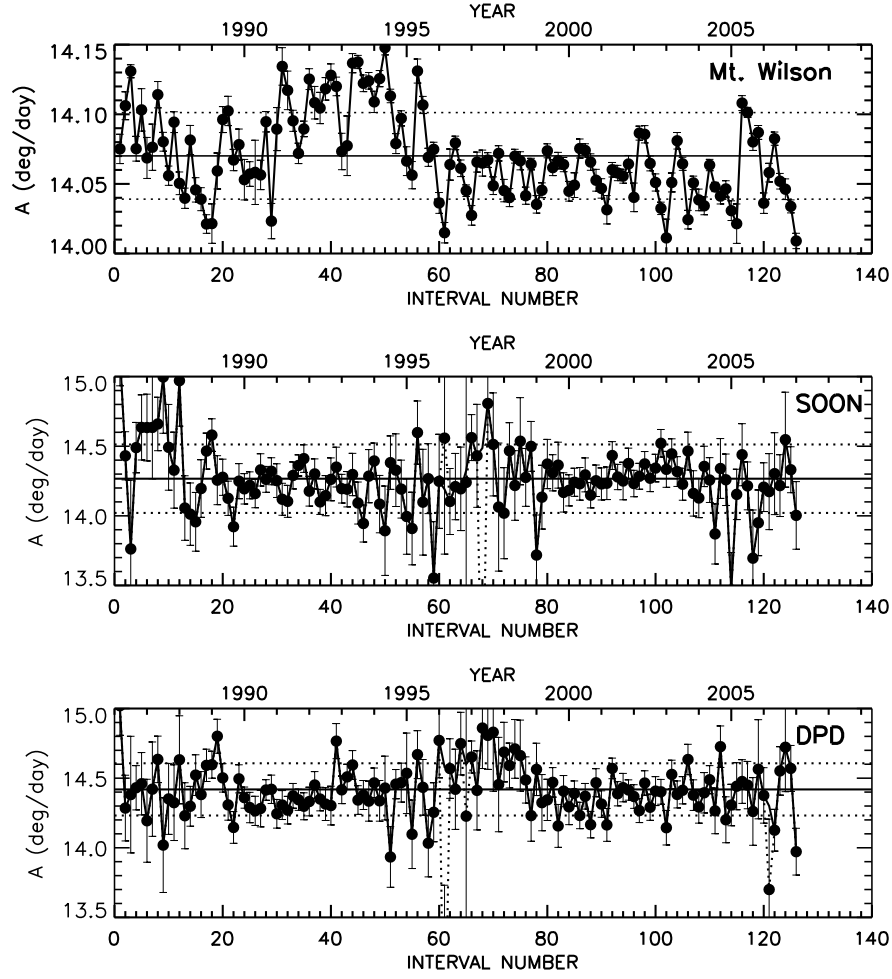


Figure 1. Plots of A in 61-day intervals determined from Mt. Wilson Doppler-velocity data (upper panel), SOON (middle panel), and DPD (lower panel) sunspot-group data during the period 1986–2007 *versus* time (interval numbers). Error-bar represents the standard error in case of the velocity data and standard deviation (σ) in case of the sunspot-group data. The horizontal continuous line represents the mean and the horizontal dotted lines indicate the corresponding root-mean-square deviations. In case of A determined from the sunspot-group data, the values whose σ values exceeded by 2.6 times the corresponding median value are replaced with the average of the corresponding values and their respective two neighbors. The continuous curve represents the corrected data, and the original data points are connected by the dotted curve.

spectra are still consistent with the conclusions and discussion in that early article.

Figures 2 and 3 show a considerable similarity in both the FFT and MEM power spectra of the A coefficient determined from the velocity and the sunspot-group data. That is, in the vicinity (within the uncertainty limit) of most of the peaks in the FFT and MEM spectra there are peaks in the corresponding

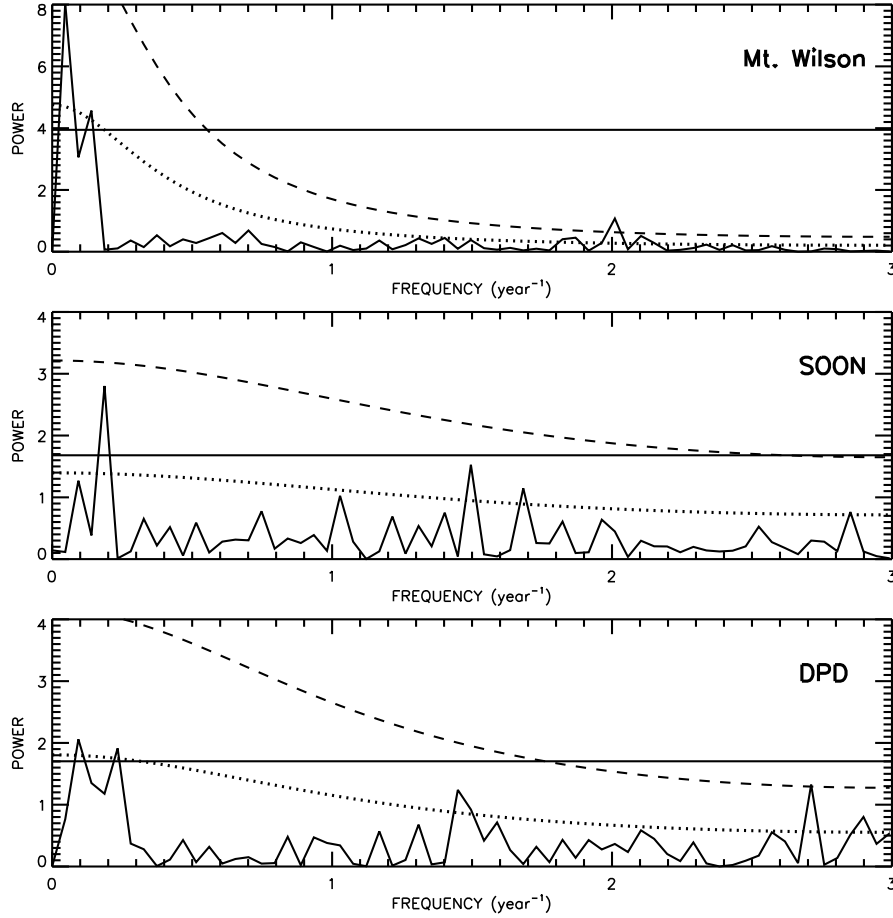


Figure 2. The upper panel shows the FFT spectrum of A computed for the 61-day intervals of the corrected Mt. Wilson Doppler-velocity data after subtracting the cosine fit of the one-year period. The middle and lower panels are the FFT spectra of A determined from the corrected 61-day binned SOON and DPD sunspot-group data, respectively. The continuous horizontal lines are drawn at the 3σ levels of the power in the respective spectra. The dotted and dashed curves represent the mean and 90% significant red-noise spectra of A determined from the Mt. Wilson velocity-data (the assumed lag-1 auto-correlation $\alpha = 0.653$), SOON ($\alpha = 0.165$) and DPD ($\alpha = 0.288$) sunspot-group data.

spectra of A determined from the sunspot-group data. Only a few peaks present in one spectrum are absent in the other (this can be seen easily in the MEM spectra.) The correlation between the FFT spectra of the velocity data and SOON sunspot-group data is poor (3.3%), but the correlation between the FFT spectra of velocity data and DPD sunspot-group data is much higher (35%), and it is even better than the correlation (27%) between the FFT spectra of SOON and DPD sunspot-group data. The values of the correlations between the corresponding MEM spectra are found to be 23%, 25%, and 20%, respectively. The 2.1-year and 156-day peaks are present in the spectra of the velocity data, but seem to be absent from the spectra of the sunspot-group data. The 9–13

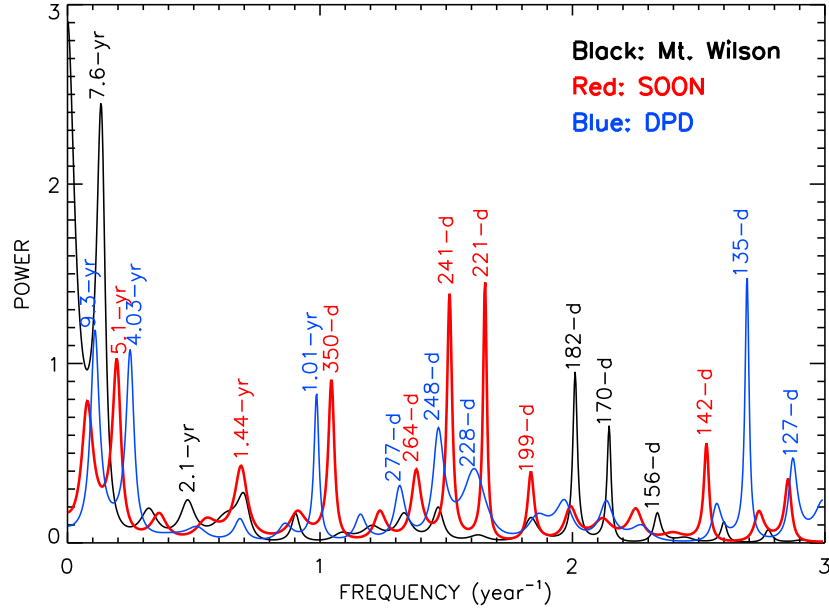


Figure 3. MEM power spectra of A determined from the corrected Mt. Wilson Doppler-velocity data, SOON, and DPD sunspot-group data shown in Figure 1. The value of the determined period is given for each well-defined peak.

year, 350-day – 1.01-year, 221 – 228 day, ~ 135 -day, and ~ 127 -day peaks seem to be present in the spectra of the sunspot-group data, but not in the spectra of the velocity data. Overall, the spectra of the velocity and the sunspot-group data show periodicities of 5–7 years, ~ 1.4 years, 241–248 days, ~ 199 days, ~ 182 days, and 142–155 days in the solar equatorial rotation rate (the 182-day periodicity may be caused by the seasonal effect). However, except for the 5–7-year peaks, none of other peaks are significant in the FFT spectra at a 95% confidence level.

The values of the assumed lag-1 auto-correlation (α) of the red-noise models (Torrence and Compo, 1998) of the FFT spectra of the velocity and the sunspot group-data are 0.653 and 0.165, respectively (note that $\alpha = 0$ yields white-noise spectrum). In the case of the red-noise model, for a peak to be significant at a given significance level, a higher value of the power is required than for the white-noise background spectrum (Torrence and Compo, 1998). Except for the ~ 135 -day peak in the FFT spectrum of DPD sunspot-group-data, none of the other peaks is significant at a 90% confidence level in the red-noise model of the FFT spectrum of A . Only a peak at a period of ~ 182 day period is significant in the FFT spectrum of the velocity data. Therefore, no significant periodicities are detected from this analysis.

Figure 4 seems to suggest a strong periodicity of ~ 248 days (~ 0.69 years) in the time series of A determined from both the Mt. Wilson Doppler-velocity data and sunspot-group data, but occurring at different temporal epochs. That

is, this periodicity appears in the velocity data around 1991, while it appears in the sunspot-group data around 1995. This periodicity appears in both SOON and DPD sunspot-group data around 1986 and 2005 (it appears to be strong in SOON data). That is, this periodicity seems to exist at ~ 11 -year intervals in the A coefficient of the sunspot-group data, mainly near the minimum epochs of solar cycles. Figure 4 also shows a ~ 182 -day periodicity around 1987, 1995, and 2005 in both the velocity and the sunspot-group data. A weaker ~ 1.4 -year signal is present in the spectrum of the velocity data during the period 1989–1995, while in the spectrum of A determined from the sunspot-group data the same periodicity is present around 1988 and during 1997–1999. A weak and slightly longer (1.6–1.7-year) periodicity seems to exist after 2002 in the velocity data, while a ~ 5 -year periodicity appears in the A time series of the sunspot groups almost throughout the period 1986–2006. There is also a weak signal with a 3–4-year periodicity in the sunspot-group data mainly during Cycle 22. The temporal dependence of the ~ 7.6 -year periodicity found in the FFT and MEM analyses of the velocity data is not detected here.

Figure 5 shows the variations in A determined from the combined GPR and SOON sunspot-group data during the period 1974–2014 and DPD sunspot-group data during the same period. This figure shows that during 1974–1976 there is a large difference in the values of A determined from the combined GPR and SOON sunspot-group data and the DPD data which is most likely due to the large uncertainties in A during periods of minimum solar activity. In particular, during the last deep and prolonged minimum (between Cycles 23 and 24) the values are much more uncertain and the difference between the values of the SOON and DPD sunspot-group data seem to be even larger. The correlation between the variations in A determined from the combined GPR and SOON data and DPD data is found to be only 14%.

Figure 6 shows the FFT and MEM power spectra, and Figure 7 shows the Morlet wavelet spectra of A determined from the combined GPR and SOON data and DPD data shown in Figure 5. Most of the peaks seen in the FFT and MEM spectra of A determined from the sunspot group-data during 1986–2007 shown in Figures 2 and 3 are also present in the spectra shown in Figures 6 and 7. In the FFT spectrum of A determined from the DPD sunspot-group data during 1974–2014 the peak at frequency $1/273 \text{ day}^{-1}$ is significant at a 95% confidence level in the white-noise model and at a 90% confidence level in the red-noise model. There are peaks at ~ 143 days and ~ 175 days in the combined GPR and SOON data, which are absent from DPD data. The peaks of ~ 2.1 -year and ~ 1.44 -year periodicities are poorly visible.

In Figure 7, a strong signal with a 250–270-day periodicity exists during 2007–2010 in the A data determined from the DPD sunspot-group data. Moreover, this periodicity seems to exist at regular intervals (*i.e.* at the minimum of each solar cycle). This periodicity is to some extent also present in the wavelet spectrum of the combined GPR and SOON data. In addition, these data show the existence of ~ 1.4 -year periodicity around 1990, not identifiable in the DPD data. On the other hand, there are strong signatures of this periodicity in the DPD data around 1976 and 2010, and a weaker one close to 1995. Indication of such a periodicity can be also found in the combined GPR and SOON data during

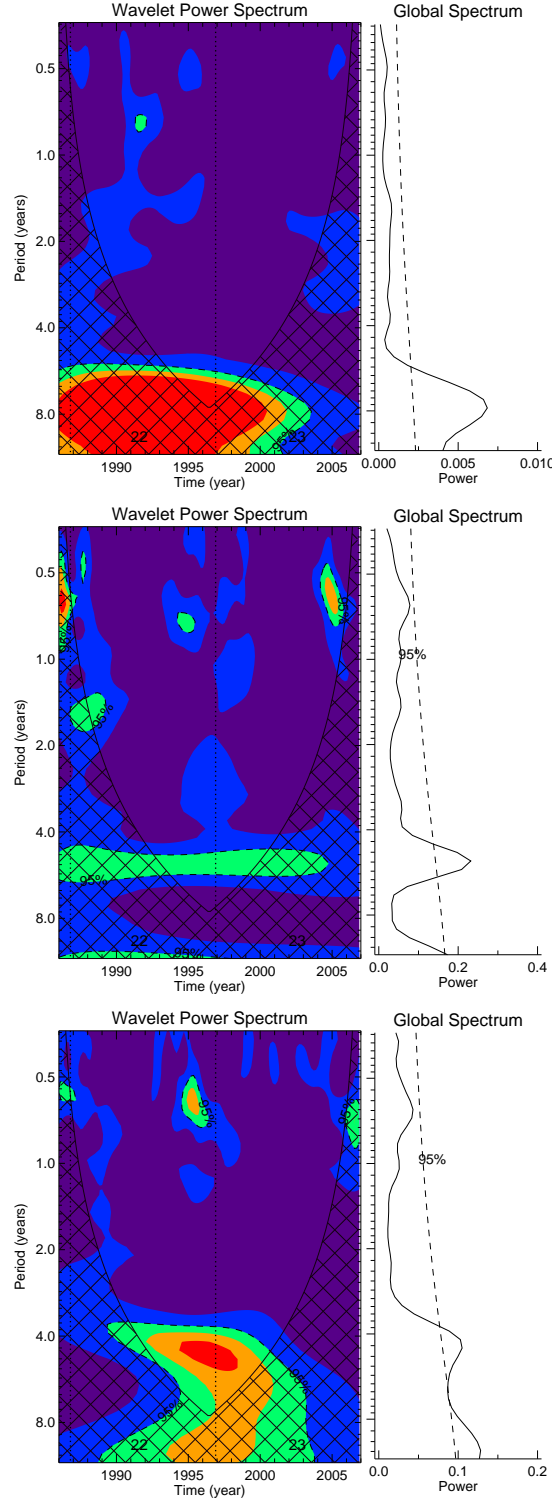


Figure 4. Wavelet power spectra and global spectra of A determined from the time series of the corrected Mt. Wilson Doppler-velocity data (upper panel), SOON (middle panel) and DPD (lower panel) sunspot-group data. The wavelet spectra are normalized by the variances of the corresponding time series. The shadings are at the normalized variances of 1.0, 3.0, 4.5, and 6.0. The dashed curves represent the 95% confidence levels deduced by assuming a white-noise process. The cross-hatched regions indicate the cone of influence where edge effects become significant (Torrence and Compo, 1998). The dotted vertical lines indicate the minima of the solar cycles. The Waldmeier number of the solar cycle is also given.

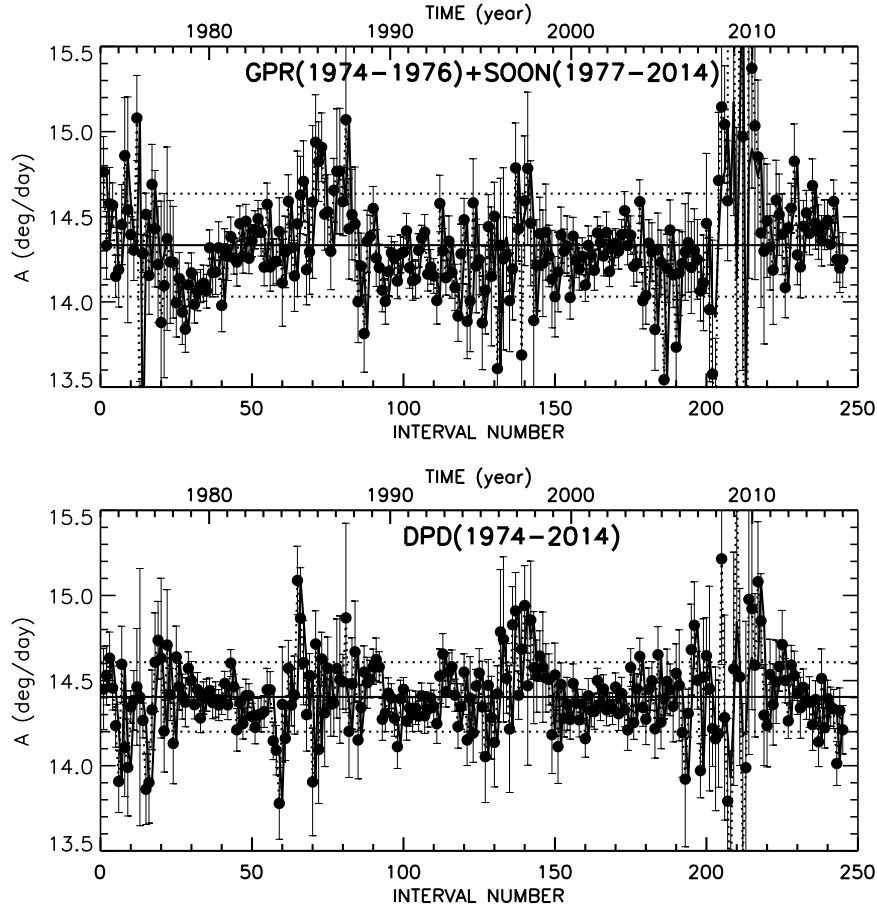


Figure 5. The lower and upper panels show plots of A in 61-day intervals determined from the DPD (1974–2014) and the combined GPR (1974–1976) and SOON (1977–2014) sunspot-group data *versus* time (interval numbers), respectively. Error-bars represent the corresponding $1.0\text{-}\sigma$ values. The horizontal continuous line represents the mean and the horizontal dotted lines indicate the corresponding root-mean-square deviations. The values whose σ values exceeded by 2.6 times the corresponding median value are replaced with the average of the corresponding values and their respective two neighbors. The continuous curve represents the corrected data, and the original data points are connected by the dotted curve.

1976. There is a very strong signal of a periodicity of ~ 2 years in the DPD data around 1985, while there is no signal of this periodicity in the combined GPR and SOON data. There are weak signals (not well resolved) of this periodicity close to 1976 and close to 2008 in the combined GPR and SOON data. A ~ 4.5 -year periodicity exists during the period 1980–2012 in the combined GPR and SOON data, while the DPD data show a periodicity ~ 4 -years during 1980–1998 and a strong periodicity of ~ 5.4 years from 1998 onward. A 10–12-year periodicity seems too strong in the combined GPR and SOON data throughout 1974–2014. A similar periodicity seems to exist in the DPD data, but the main portion of the corresponding power is within the cross-hatched regions where edge effects

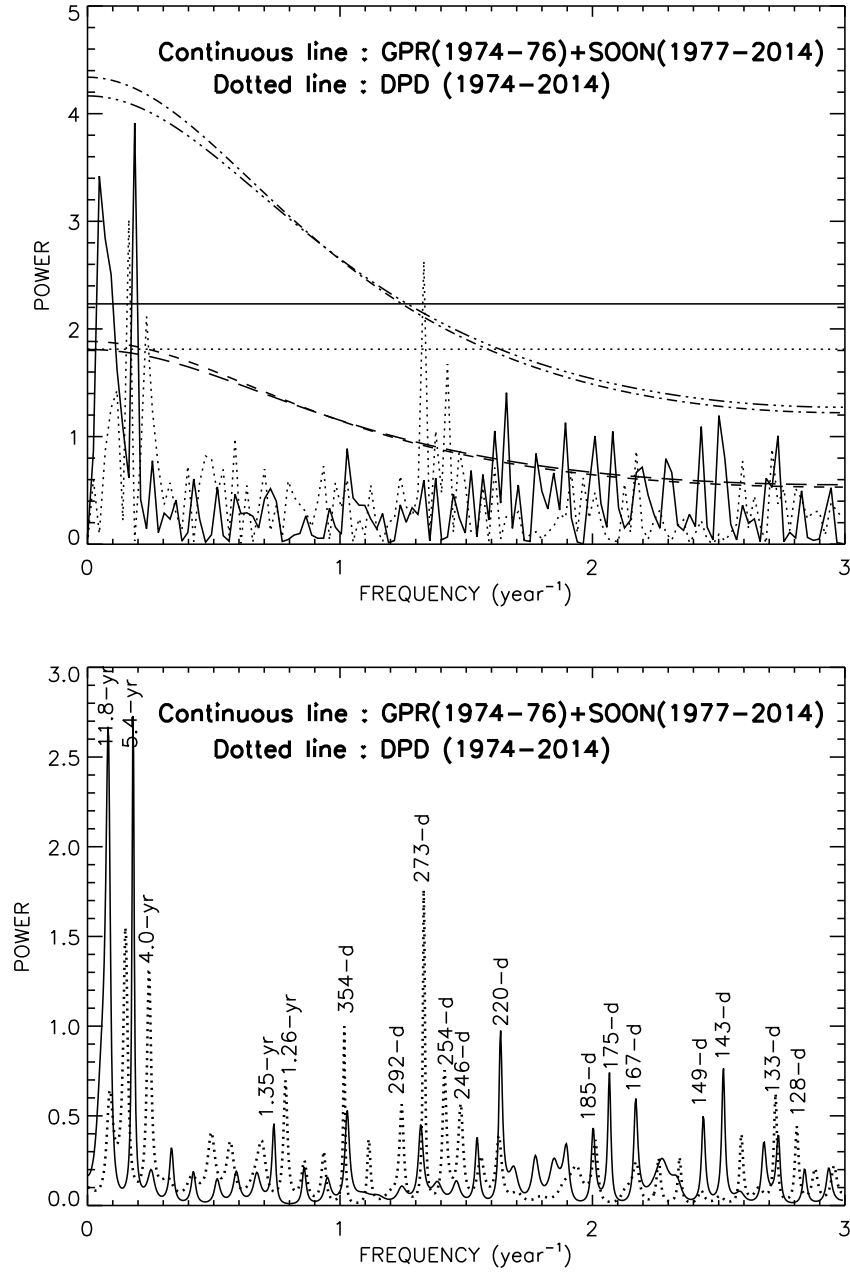


Figure 6. FFT (upper panel) and MEM (lower panel) power spectra of A determined from the corrected sunspot-group data shown in Figure 5. In the upper panel the long-dashed and long dashed-dotted curves represent the mean and 90% confidence level red-noise spectra of A determined from the combined GPR and SOON sunspot-group data during 1974–2014 ($\alpha = 0.3065$), and the dashed and dash-dotted curves represent the corresponding spectra determined from DPD sunspot-group data 1974–2014 ($\alpha = 0.2870$).

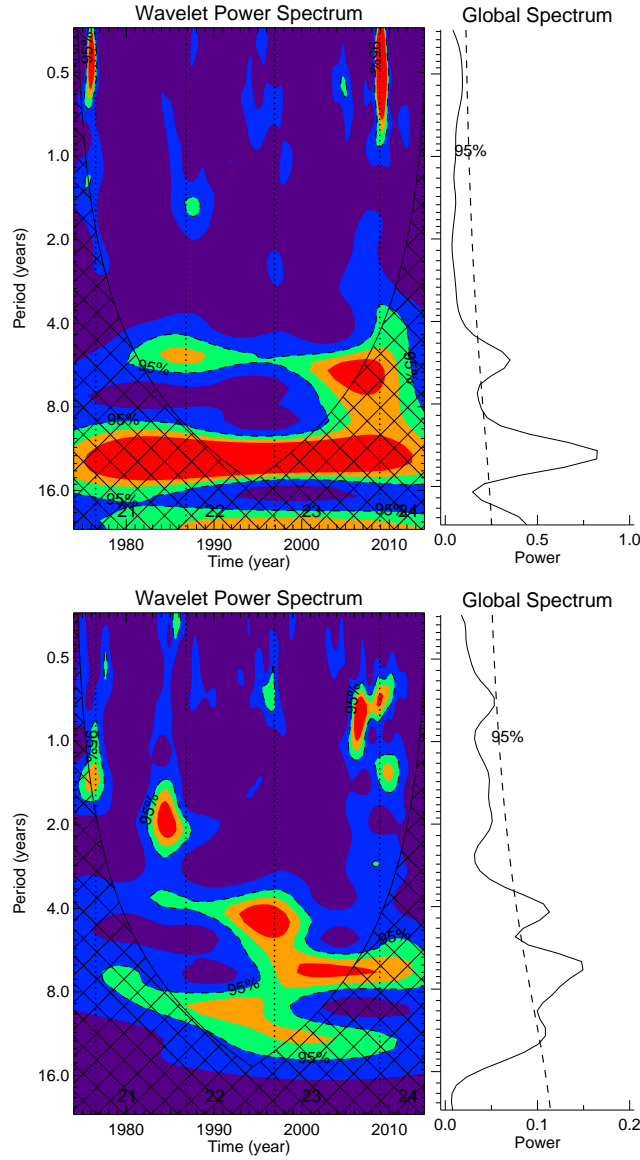


Figure 7. Wavelet power spectra and global spectra of A determined from the combined GPR+SOON (upper panel) and DPD (lower panel) sunspot-group data shown in Figure 5. The wavelet spectra are normalized by the variances of the corresponding time series. The shadings are at the normalized variances of 1.0, 3.0, 4.5, and 6.0. The dashed curves represent the 95% confidence levels deduced by assuming a white-noise process. The cross-hatched regions indicate the cone of influence where edge effects become significant (Torrence and Compo, 1998). The dotted vertical lines indicate the minima of the solar cycles. The Waldmeier number of the solar cycle is also given.

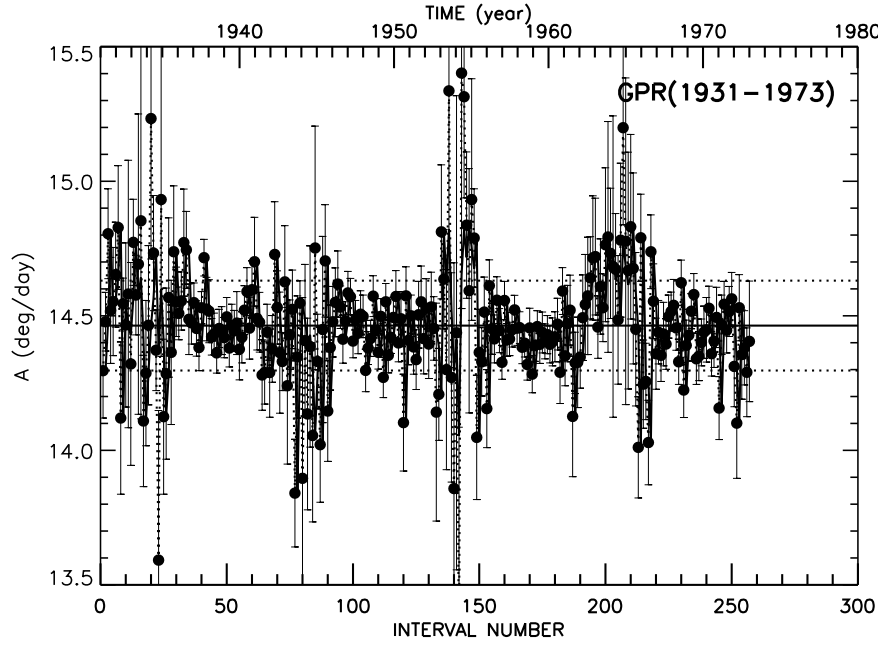


Figure 8. Plot of A in 61-day intervals determined from the GPR sunspot-group data during 1931–1973 *versus* time (interval numbers). Error bars represent the corresponding $1.0\text{-}\sigma$ values. The horizontal continuous line represents the mean and the horizontal dotted lines indicate the corresponding root-mean-square deviations. The values whose σ values exceeded by 2.6 times the corresponding median value are replaced with the average of the corresponding values and their respective two neighbors. The continuous curve represents the corrected data, and the original data points are connected by the dotted curve.

are significant. A similar behavior can also be seen in the wavelet spectrum of the DPD data.

Figure 8 shows the variations in the A coefficient determined from the 61-day binned GPR sunspot-group data during the period 1931–1973. Figures 9 and 10 show the corresponding FFT, MEM, and Morlet wavelet power spectra. Both the FFT and MEM spectra show similar sharp and well-defined peaks. The periodicity of ~ 1.56 years is clearly visible in both of these spectra, above the 99% confidence level using the white-noise model and above the 90% using the red-noise model. The FFT and MEM spectra also show a significant ~ 17.5 -year peak, which reaches the 99% confidence level (white-noise model) in the FFT spectrum. This periodicity was also revealed in an earlier study of the sunspot group data by (Javaraiah, 2005). As in Figures 2, 3, and 6, many other peaks are present in both spectra. The periods at 192 days, 156 days, and 110 days periods are significant at a 90% confidence level in the red-noise model. It should be noted that, as previously mentioned, most of the known short-term periodicities, ≤ 5 year, in solar activity seem to appear only from time to time, *i.e.* they are highly intermittent. The wavelet spectrum (Figure 10) shows a strong signal with a 1.56-year periodicity around 1933 and 1955 that are also visible in the corresponding FFT and MEM power spectra (Figure 9), and a strong ~ 1.8 -year periodicity

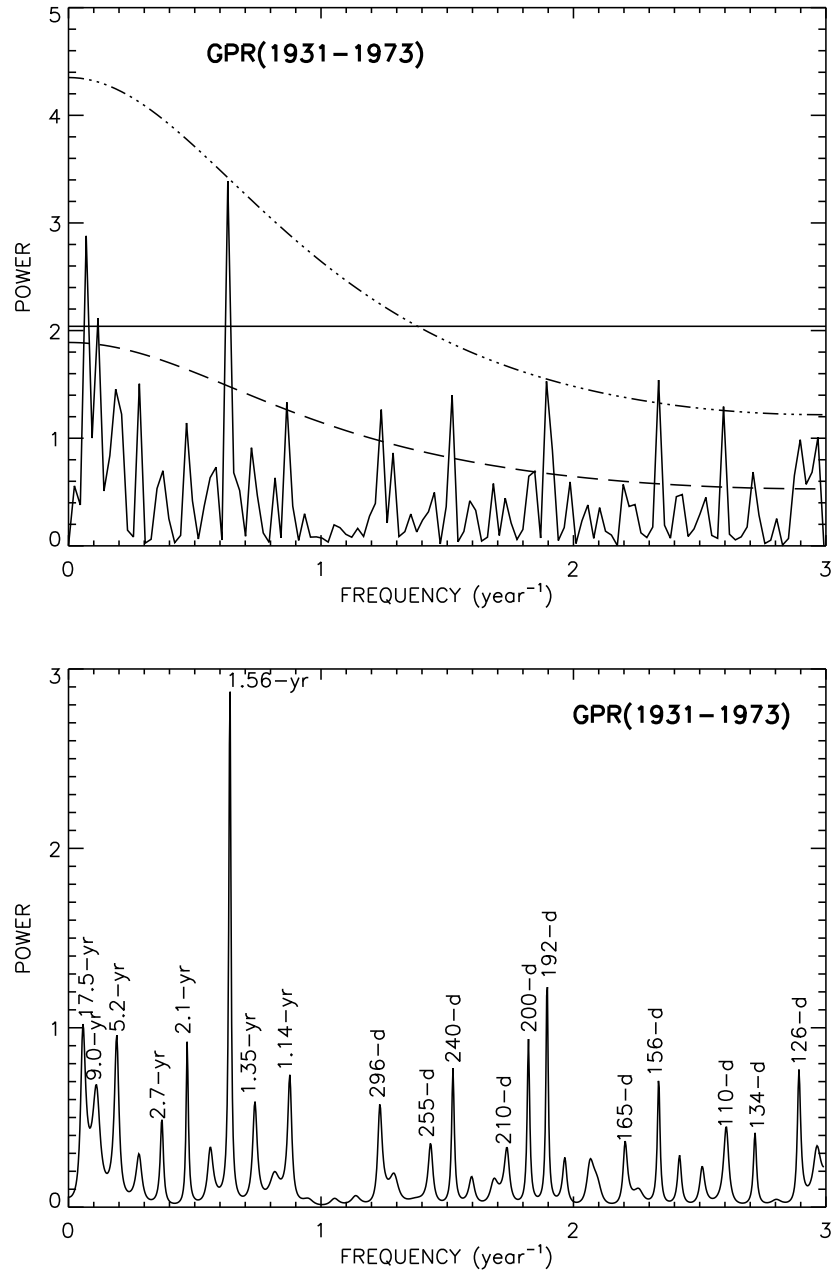


Figure 9. FFT (upper panel) and MEM (lower panel) power spectra of A determined from the corrected sunspot-group data shown in Figure 8. In the upper panel the long dashed and long dash-dotted curves represent the corresponding mean and 90% confidence level red-noise spectra ($\alpha = 0.308$).

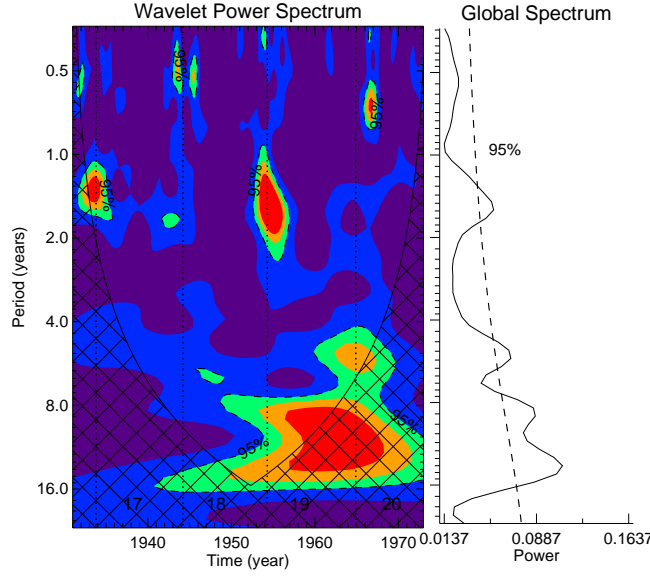


Figure 10. Wavelet power spectra and global spectra of A determined from the GPR sunspot-group data shown in Figure 8. The wavelet spectrum is normalized by the variance of the corresponding time series. The shadings are at the normalized variances of 1.0, 3.0, 4.5, and 6.0. The dashed curves represent the 95% confidence levels deduced by assuming a white-noise process. The cross-hatched region indicates the cone of influence where edge effects become significant (Torrance and Compo, 1998). The dotted vertical lines indicate the minima of the solar cycles. The Waldmeier number of the solar cycle is also given.

around 1943. The periodicity at ~ 296 days is clearly present near 1967, and the ~ 200 -day periodicity seems to appear around 1932, 1944, and 1946. A weak signal with a 4–6-year periodicity throughout 1931–1973 becomes very strong around 1965. The 10–17-year periodicity seems to be present only after 1940, particularly during the 1950–1970 time interval.

Overall, the wavelet spectra of A determined from the sunspot group data suggest that most of the short-term periods in A appear around the minimum of the solar cycles, where the uncertainties in the values of A are substantial. The timing of the short-term periods in the velocity and sunspot-group data also disagrees considerably. The periods found in the velocity data before 1995 may be artifacts due to frequent changes in the Mt. Wilson spectrograph instrumentation (Javaraiah *et al.*, 2009). The abnormal longitudinal drifts ($> 3^\circ \text{ day}^{-1}$) of sunspot groups (Javaraiah, 2013) were not excluded from our analysis. This may contribute to the uncertainties in the periods found in A from the sunspot-group data. In addition, the evolution of sunspot groups may also play a significant role. These problems raise some questions about the existence and/or significance of the short-term periodicities found here in the A time series. However, these periodicities have also been found in other solar activity indices. For example, Gurgenchvili *et al.* (2016) have detected similar signatures in selected datasets during a few years of the solar cycles and have shown that their length and level

of significance vary with time. This suggests that the short-term periodicities found in our investigation are worth further study to prove their existence.

4. Conclusions and discussion

Our analysis indicates that short-term variations in the solar equatorial rotation rate are highly intermittent in nature. A period of ~ 250 days is found in the variations of the equatorial rotation rate determined from both the Mt. Wilson Doppler-velocity data and the sunspot-group data. In the equatorial rotation rate of sunspot groups a strong ~ 1.4 -year period is found around 1956, and weak signals of this period are seen also around 1935, 1976, 1986, and 1996. The velocity data show a weak signature of this periodicity during 1990–1995 in these measurements. A strong periodicity of ~ 1.6 years is found during 1933 and 1955 in the equatorial rotation rate of sunspot groups. A weaker signal of the same period is seen around 1976, while a strong ~ 1.8 -year periodicity is found around 1943. There are indications that periods of ~ 5.4 years, ~ 11 years, and ~ 17.5 years exist in the equatorial rotation rate of sunspot groups, while a ~ 7.6 -year period is found only in the velocity data. In the sunspot data there is also a suggestion of several other short-term periods, *viz.*, ~ 182 days (around 1987, 1995, and 2005), ~ 200 days (around 1932, 1944, and 1946), ~ 1 year, ~ 2 years, ~ 4 years, *etc.* However, the actual existence of most of the short-term periodicities found here needs to be confirmed because they might be artifacts of the large uncertainties in the data that are due to various factors, for example, frequent changes in the Mt. Wilson spectrograph instrumentation, abnormal longitudinal drifts ($> 3^\circ \text{ day}^{-1}$) of sunspot group, evolution of sunspot groups.

Since on average the rotation rate of sunspot groups (in general magnetic regions) represents the rotation rate of somewhat deeper layers of the Sun depending upon the sizes of the groups (Foukal, 1972; Gilman and Foukal, 1979; Nesme-Ribes, Ferreira, and Mein, 1993; Howard, 1996; Javaraiah and Komm, 1999; Javaraiah and Gokhale, 1997b; Hiremath, 2002; Sivaraman *et al.*, 2003; Javaraiah, 2013), the ~ 1.4 -year periodicity in the equatorial rotation rate of sunspot groups may be related to the known 1.3-year periodicity in the low-latitude rotation rate at the base of the convection zone (Howe *et al.*, 2000). The physical connection between a periodicity in the solar rotation and the corresponding periodicity in solar activity may be explained as follows: the variation in the emerging magnetic flux on the Sun could be affected by the corresponding modulation in the solar rotation, through the effect of the Coriolis force on the emerging magnetic flux. However, the causes of the variation in the solar rotation are unknown. One possible explanation is that the variation in solar differential rotation may be caused by Rossby-type waves as discussed by Ward (1965), Kuhn, Albrecht, and Dickie (1998), and Knaack, Stenflo, and Berdyugina (2005). Another possibility is that short-term periodicities may be caused by internal gravity waves as discussed by Wolff (1983), or that the source of the perturbation may come from outside the Sun, *i.e.* from solar system dynamics (Wood and Wood, 1965; Javaraiah and Gokhale, 1995; Zaqarashvili, 1997; Juckett, 2000; Javaraiah, 2003, 2005; Wilson, Carter, and Waite, 2008; Gokhale, 2010;

Wolff and Patrone, 2010; Tan, 2011; Cionco and Compagnucci, 2012; Abreu *et al.*, 2012; Wilson, 2013; Chowdhury *et al.*, 2016 and references therein). In the latter case, it is interesting to note that the periodicities of ~ 250 days and ~ 1.6 years are approximately the orbital period of the planet Venus and its modulation caused by relative position of Earth and Jupiter. All these are important planets because their tidal forces are relatively strong. Moreover, the combined effect of these planets seems to be strongest at the times that are at or near solar cycle minima (Wilson, 2013). This might be a reason for the short-term periodicities found around solar cycle minima. Nevertheless, we cannot rule out the possibility that the variations in the rotation rate of sunspot groups simply represent the variations in the sunspot emergence along with the group size and longitudinal displacement.

The periodicities of 182 days and one year periodicities are mostly artifacts of the seasonal effects, but they may be also related to the relative positions of Earth and Jupiter. Most of the short-term periodicities may be related to relative positions of Mercury, Venus, Earth, Mars and Jupiter. However, mere matching of periodicities is no guaranty for a physical relationship. The periodicity of ~ 2 years may be related to the well-known quasi-biennial oscillation of the solar activity, and it may be related to the configuration of Earth, Mars, and Jupiter (the Earth-Mars synodic period is 2.136 years). Any pair of planets will be in alignment and produce relatively high suntides periodically at intervals equal to one-half their mean synodic period. Therefore, the origin of other relatively long-term periodicities can also be explained on the basis of alignments of Venus and Earth, relative to the positions of Jupiter (also see Wilson, 2013). That is, the periodicities of ~ 5.4 years, ~ 7.6 years, and ~ 17.5 years found here may be 3.5, 4.5, and 11 times the synodic period (1.597 year) of Venus and Earth. In an earlier analysis the average solar cycle variation of the equatorial rotation rate was found to be strong during an odd-numbered solar cycle and the variation was weak or absent during an even-numbered solar cycle (Javaraiah, 2003). The ~ 17.5 -year period may be related to this property. The peak of the ~ 7.6 -year period in the power spectra of the velocity data may be an artifact of the large difference in these data before and after 1995. However, a 7-year periodicity is known to exist in the solar rotation rate derived from Ca II K plage data during the period 1951–1981 (Singh and Prabhu, 1985). The origin of all of the detected periodicities still needs to be established.

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References

- Abreu, J.A., Beer, J., Ferriz-Mas, A., McCracken K.G., Steinhilber, F.: 2012, *Astron. Astrophys.* **548**, A88. DOI: 10.1051/0004-6361/20219997

- Bai, T.: 2003, *Astrophys. J.* **591**, 406. DOI: 10.1086/375295
- Bai, T., and Sturrock, P.A.: 1991, *Nature* **350**, 141. DOI: 10.1038/350141a0
- Berryman, J.G.: 1978, *Geophys.* **43**, 1384. DOI: 10.1190/1.1440902
- Bouwer, S.D.: 1992, *Solar Phys.* **142**, 365. DOI: 10.1007/BF00151460
- Brault, J.W., White, O.R.: 1971, *Astron. Astrophys.* **13**, 169.
- Chowdhury, P., Khan, M., Ray, P.C.: 2009, *Mon. Not. Roy. Astron. Soc.* **392**, 1159. DOI: 10.365-2966-2008.14117.x
- Chowdhury, P., Gokhale, M.H., Singh, J., Moon, Y.-J.: 2016, *Astrophys. Space Sci.*, **361**, ID 54, 17pp. DOI: 10.1007/s10509-015-2641-8
- Cionco, R.G., Compagnucci, R.H.: 2012, *Adv. Space Res.* **50**, 1434. DOI: 10.1016/j.asr.2012.07.013
- Foukal, P.V.: 1972, *Astrophys. J.* **173**, 439. DOI: 10.1086/151435
- Gilman, P.A., Foukal, P.V.: 1979, *Astrophys. J.* **229**, 1179. DOI: 10.1086/157052
- Gokhale, M.H.: 2010, In: S.S. Hasan and R.J. Rutten (eds.) Magnetic Coupling between the Interior and Atmosphere of the Sun, *Astrophysics and Space Science Proceedings*, Springer-Verlag Berlin Heidelberg, 494. DOI: 10.1007/978-3-642-02859-5_65
- Gurgenashvili, E., Zaqarashvili, T.V., Kukhianidze, V., Oliver, R., Ballester, J.L., Ramishvili, G., Shergelashvili, B., Hanslmeier, A., Poedts, S.: 2016, *Astrophys. J.* **826**, 55. DOI: 10.3847/0004-637X/826/1/55
- Györi, L., Baranyi, T., Ludmány, A.: 2010, in *Proc. Intern. Astron. Union 6, Sympo. S273*, **2011**, 403. DOI: 10.1017/s174392131101564X
- Hathaway, D.H.: 2015, *Living Rev. Solar. Phys.* **12**, No.4. DOI: 10.1007/lrsp-2015-4
- Hathaway, D.H., Choudhary, D.P.: 2008, *Solar Phys.* **250**, 269. DOI: 10.1007/s11207-008-9226-4
- Hathaway, D. H., Nandy, D., Wilson, R. M., Reichmann, E. J., 2003, *Astrophys. J.* **589**, 665. DOI: 10.1086/374393
- Hiremath, K.M.: 2002, *Astron. Astrophys.* **386**, 674. DOI: 10.1051/0004-6361:20020276
- Howard, R.F.: 1996, *Ann. Rev. Astron. Astrophys.* **34**, 75. DOI: 10.1146/annurev.astro.34.1.75
- Howe, R., Christensen-Discard, J., Hill, F., Komm, R.W., Larsen, R.M., Scour, J., Thompson, M.J., Toomre, J.: 2000, *Science* **287**, 2456. DOI: 10.1126/science.287.5462.2456
- Javaraiah, J.: 2003, *Solar Phys.* **212**, 23. DOI: 10.101023/A:1022912430585
- Javaraiah, J.: 2005, *Mon. Not. Roy. Astron. Soc.*, **362**, 579. DOI: 10.1111/j.1365-2966.2005.09403.x
- Javaraiah, J.: 2011, *Adv. Space Res.* **48**, 1032. DOI: 10.1016/j.asr.2011.05.004
- Javaraiah, J.: 2013, *Solar Phys.*, **287**, 197. DOI: 10.1007/s11207-013-0345-1
- Javaraiah, J., Bertello, L., Ulrich, R.K.: 2005, *Astrophys. J.* **626**, 579. DOI: 10.1086/429898
- Javaraiah, J., Gokhale, M.H.: 1995, *Solar Phys.* **158**, 173. DOI: 10.101023/BF00680841
- Javaraiah, J., Gokhale, M.H.: 1997a, *Solar Phys.* **170**, 389. DOI: 10.1023/A:1004928020737
- Javaraiah, J., Gokhale, M.H.: 1997b, *Astron. Astrophys.* **327**, 795.
- Javaraiah, J., Komm, R.W.: 1999, *Solar Phys.* **184**, 41. DOI: 10.1023/A:1005028128077
- Javaraiah, J., Ulrich, R.K., Bertello, L., Boyden, J.E.: 2009, *Solar Phys.* **257**, 61. DOI: 10.1007/s11207-009-9342-9
- Juckett, D.: 2000, *Solar Phys.* **191**, 201. DOI: 10.1023/A:1005226724316
- Kane, R.P.: 2003, *J. Atmos. Solar-Terr. Phys.* **65**, 979. DOI: 10.1016/S1364-6826(03)00117-2
- Kilcik, A., Özgüç, A., Yurchyshyn, V.B., Rozelot, J.P.: 2014, *Solar Phys.* **289**, 4365. DOI: 10.1007/s11207-014-0580-0
- Knaack, R., Stenflo, J.O., Berdyugina, S.V.: 2005, *Astron. Astrophys.* **438**, 1067. DOI: 10.1051/0004-6361:2004209
- Krivova, N.A., Solanki, S.K.: 2002, *Astron. Astrophys.* **394**, 701. DOI: 10.1051/0004-6361:20021063
- Kuhn, K.G., Albrecht, K.G., Dickie, R.H.: 1998, *Science* **242**, 908. DOI: 10.1126/science.242.4880.908
- Lean, J.L., Brueckner, G.E.: 1989, *Astrophys. J.* **337**, 568. DOI: 10.1086/167124
- Nesme-Ribes, E., Ferreira, E.N., Mein, P.: 1993, *Astron. Astrophys.* **274**, 563.
- Obridko, V.N., Shelting, B.D.: 2000, *Astron. Rep.*, **44**, 262. DOI: 10.1134/1.163849
- Obridko, V.N., Shelting, B.D.: 2007, *Adv. Space Res.* **40**, 1006. DOI: 10.1016/j.asr.2007.04.105
- Özgüç, A., Atac, T., Rybák, J.: 2003, *Solar Phys.* **214**, 375. DOI: 10.1023/A:1024225802080
- Pap, J., Bouwer, S.D., Tobiska, W.K.: 1990, *Solar Phys.* **129**, 165. DOI: 10.1007/BF00154372
- Richardson, J.D., Paularena, K.I., Belcher, J.W., Lazarus, A.J.: 1994, *Geophys. Res. Lett.* **21**, 1559. DOI: 10.1029/94GL01076

- Rieger, E., Kanbach, G., Reppin, C., Share, G.H., Forrest, D.J., Chupp, E.L.: 1984, *Nature* **312**, 623. DOI: 10.1038/312623a0
- Scafetta, N., Willson, R.C.: 2013, *Pattern Recogn. Phys.* **1**, 123. DOI: 10.5194/prp-1-123-2013
- Singh, J., Prabhu, T.P.: 1985, *Solar Phys.* **97**, 203. DOI: 10.1007/BF00152989
- Sivaraman, K.R., Sivaraman, H., Gupta, S.S., Howard, R.: 2003, *Solar Phys.* **214**, 65. DOI: 10.1086/148143/A:1024075100667
- Tan, B.: 2011, *Astrophys. Space Sci.* **332**, 65. DOI: 10.1007/s10509-010-0496-6
- Torrence, Ch., Compo, G.P.: 1998, *Bull. Am. Meteor. Soc.* **79**, 61. DOI: 10.1175/1520-0477(1998)079<0061:APGTWA>2.0.CO;2
- Ulrich, R.K. 2001, *Astrophys. J.* **560**, 466. DOI: 10.1086/322524
- Ulrych, T.J., Bishop, T.N.: 1975, *Rev. Geophys. Space Phys.* **13**, 183. DOI: 10.1029/RG013i001p00183
- Ward, F.: 1965, *Astrophys. J.* **141**, 534. DOI: 10.1086/148143
- Wilson, I.R.G.: 2013, *Pattern Recogn. Phys.* **1**, 147. DOI: 10.5194/prp-1-147-2013
- Wilson, I.R.G., Carter, B.D., Waite, I.A.: 2008, *Publ. Astron. Soc. Aust.* **25**, 85. DOI: 10.1071/AS06018
- Wolff, C.L.: 1983, *Astrophys. J.* **264**, 667. DOI: 10.1086/160640
- Wolff, C.L., Patrone, P.N.: 2010, *Solar Phys.* **266**, 227. DOI: 10.1007/s11207-010-9628-y
- Wood, R.M., Wood, K.D.: 1965, *Nature* **208**, 129. DOI: 10.1038/208129a0
- Zaqarashvili, T. V.: 1997, *Astrophys. J.* **487**, 930.